# Integrated Workforce Scheduling and Routing Problem


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## Introduction

Problems combining workforce scheduling and routing are present in the daily activities of several companies. From cable television technical support to homecare, such companies need to allocate their staff (skilled technicians, nurses, care workers, etc.) to daily activities located at different geographical locations, hence moving between locations is part of the operational problem. Since the number of tasks usually exceeds by far the number of employees, it is necessary to build an efficient schedule and assign workers in the best possible manner to have every task completed. It is also often desirable to minimize the travel distance between assignments.

This integrated problem is a difficult combinatorial optimisation problem which can be tackled by using discrete optimisation techniques, constraint programming or heuristic search methods.

## Groups of Constraints

The constraints in this integrated scheduling and routing problem can be put into three groups corresponding to each aspect of the problem:

- Scheduling – appropriate management of human resources in the production process, maintenance equipment or service task
- Routing – define a route for each human resource to visit customers
- Assignment – assign tasks within specific time with the right number of workers

Scheduling constraints:
(1) Skills and qualifications
(2) Work regulations
(3) Staff preferences
(4) Client preferences
(5) Precedence between tasks
(6) Teaming
(7) Days off and holidays

Routing constraints:
(8) Route continuity
(9) Start and end locations
(10)Time windows
(11)Sub-tour elimination
(12)Transportation modality

Assignment constraints:
(13)Task assignment
(14)Arrival time allocation
(15)Worker quantity requirement

## Ongoing Project

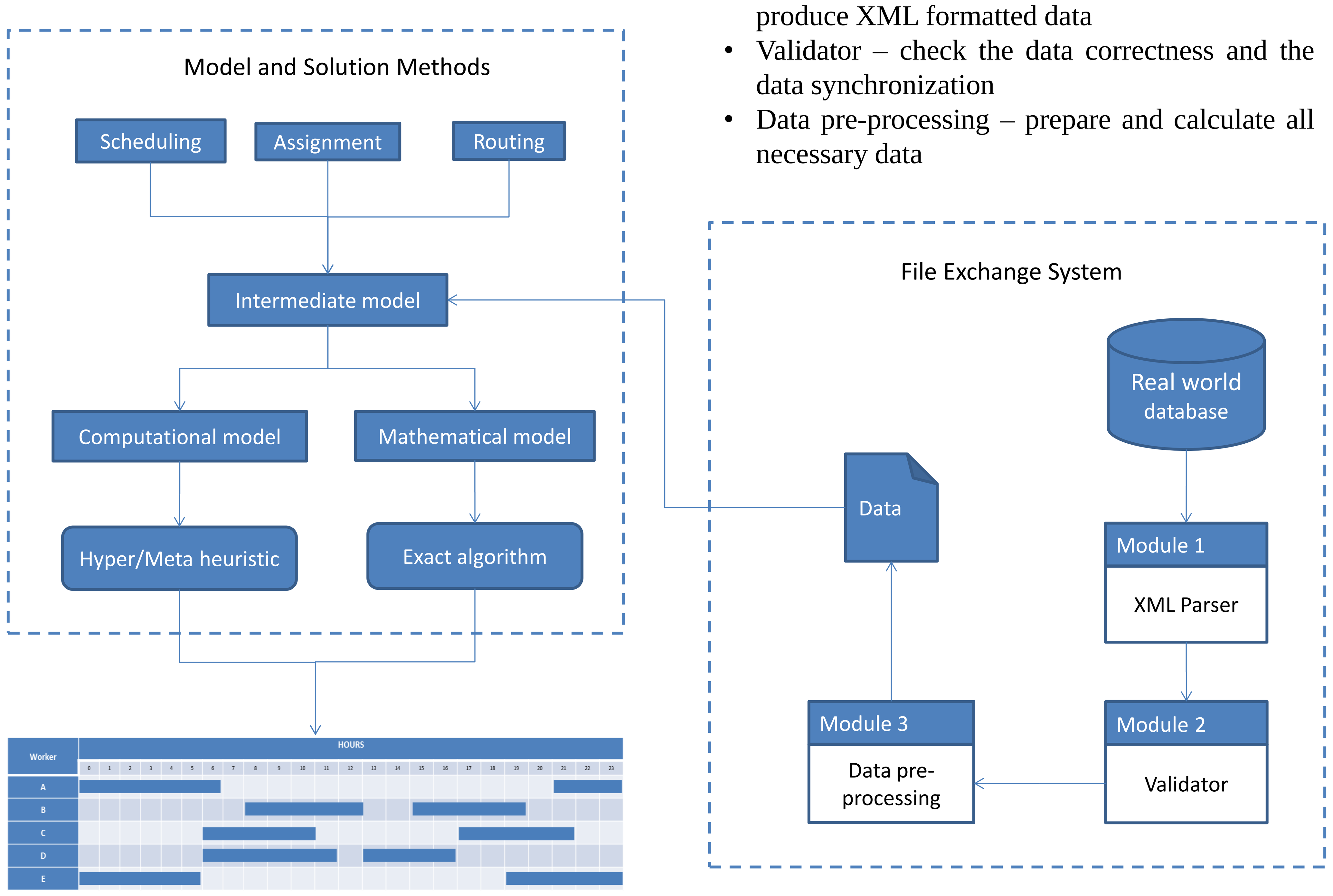
The project has been split into two major areas:

Mathematical modelling and development of solution methods

- Implementing the integrated model
- Developing solution methods

A File Exchange System is designed to be a software for creating the intermediate files between data provider and researcher. The software has three main modules. In addition, the intermediate files are designed based on an XML structure.

- XML parser – read raw data from database and produce XML formatted data
- Validator – check the data correctness and the data synchronization
- Data pre-processing – prepare and calculate all necessary data




## Mathematical Model

$$\text{Min} \sum_{c \in C} \sum_{i \in D \cup T} \sum_{j \in D' \cup T} \left( x_{i,j}^{c} d_{i,j} + x_{i,j}^{c} t_{i,j} - x_{i,j}^{c} p_{c,j} \right) \quad (3)$$

Scheduling

$$x_{i,j}^{c} r_{s,j} \le l_{s}^{c}, \qquad \forall c \in C, i \in D \cup T, j \in T, s \in S \quad (1)$$

$$\sum_{i \in D} \sum_{j \in D \cup T'} x_{i,j}^{c} q_i \le h^{c}, \qquad \forall c \in C \quad (2)$$

Routing

$$\sum_{i \in D \cup T} x_{i,j}^{c} = \sum_{i \in D' \cup T} x_{j,i}^{c}, \qquad \forall j \in T, c \in C$$

$$\sum_{i \in D' \cup T} x_{i,j}^{c} \le 1, \qquad \forall j \in D, \forall c \in C \quad (8)$$

$$\sum_{i \in D \cup T} x_{i,j}^{c} \le 1, \qquad \forall j \in D', \forall c \in C$$

$$\sum_{j \in D' \cup T} x_{k,j}^{c} \ge \sum_{j \in D' \cup T} x_{i,j}^{c}, \qquad \forall c \in C, \forall i \in T, \exists k \in D$$

$$\sum_{i \in D \cup T} x_{i,k}^{c} \ge \sum_{i \in D \cup T} x_{i,j}^{c}, \qquad \forall c \in C, \forall i \in T, \exists k \in D' \quad (9)$$

$$w_i^{L} \le a_i^{c} \le w_i^{U}, \qquad \forall i \in T, c \in C \quad (10)$$

$$x_{i,j}^{c} + x_{j,i}^{c} \le 1, \qquad \forall c \in C, \forall i, j \in T$$

$$x_{i,i}^{c} \le 0, \qquad \forall c \in C, \forall i \in T \quad (11)$$

Assignment

$$\sum_{c \in C} \sum_{i \in D \cup T} x_{i,j}^{c} = 1, \qquad \forall j \in T \quad (13)$$

$$a_j^{c} + M\left(1 - x_{i,j}^{c}\right) \ge a_i^{c} + x_{i,j}^{c} t_{i,j} + q_i, \qquad \forall c \in C, i, j \in T \quad (14)$$

## Preliminary Results

The model has been tested for correctness using sample data with 50 tasks. CPLEX found the optimal solution in around 4 minutes. However, a realistic scenario will have around 480 tasks to assign during a day, hence the computational difficulty of the realistic problems is considerable higher.

## References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## Contributions

- Introduced mathematical and computational models of the integrated problem
- Developed a file exchange system to process real-world data into the optimization model and solution methods
- Delivered solution methods which handle real world datasets
- Gained better understanding of the integrated problem and its computational difficulty

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